

Analysis and Applied Mathematics Seminar

Lagrangian solutions to the Porous Media Equation (and friends)

Matt Jacobs (Purdue)

Abstract: Many works have been devoted to understanding and predicting the time evolution of a growing population of cells (bacterial colonies, tumors, etc...). At the macroscopic scale, cell growth is typically modeled through Porous Media type equations that describe the change in cell density. While these cell growth PDEs have been studied since the 70s, our understanding is far from complete, particularly in the case where there are several distinct cell populations.

An important open question is whether it is possible for two populations that were separated at initial time to become mixed during the flow. For instance, can tumor cells get mixed into healthy cell regions?

In this talk, I will show that it is possible to construct non-mixing solutions to these equations. The key is to construct the Lagrangian flow map along the pressure gradient generated by the Porous Media Equation. The main obstruction is the fact that the pressure gradient is not sufficiently regular to apply any generic theory for Lagrangian flows. To overcome this difficulty, we develop a new argument combining features of the Porous Media Equation with the quantitative Lagrangian flow theory of Crippa and De Lellis.

Monday, April 17 at 4:00 PM in 1227 SEO