Abstract: Derived two hundred years ago, the Navier-Stokes equation (NSE) governs the motion of fluids. In 1930s, Leray established the theory of weak solutions for the NSE and raised questions, some of which still remain open and center around the well-posedness problem. In the talk, we will review some progresses in the effort to understand these classical questions. The emphasis will be on some recent results, sparked by empirical laws in physics (such as Kolmogorov’s phenomenological theory of turbulence) and techniques from other fields in mathematics (for instance the convex integration scheme). We will also discuss some ongoing interests in various problems and new perspectives opened up by these techniques.