Analysis and Applied Mathematics Seminar

Unconditional Stability of KdV-Burgers Fronts
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Abstract: \[ u_t + u u_x = \eta u_{xxx} + u_{xx} \quad \lim_{x \to \mp \infty} u = \pm 1 \] Originally proposed by Whitham as a model for the propagation of tidal bores. It was shown by Bona and Schonbek that front type traveling wave solutions exist for all \( \eta \), unique modulo translation, and are monotone for \( |\eta| \leq \frac{1}{2} \), and by Pego that such solutions are stable to small perturbations for the monotone case. We present a new stability criteria that does not require a smallness assumption on the difference between the initial data and the traveling wave, and which can be shown to hold in an open set of \( \eta \) values that includes the monotone case. This condition involves the number of bound states of a certain Schrödinger operator constructed from the front solution. We will also discuss some rigorous numerical calculations that give intervals in \( \eta \) where this spectral condition is guaranteed to hold. Joint work with Blake Barker, Vera Hur and Zhao Yang.