Algebraic Geometry Seminar

The non-Lefschetz locus, jumping lines and conics

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Abstract: An Artinian Algebra $A$ has the Weak Lefschetz Property (WLP) if there is a linear form, $\ell$, such that the multiplication map $\times \ell$ from $A_i$ to $A_{i+1}$ has maximal rank for each integer $i$. We want to study the set of linear forms for which maximal rank fails, this is called the non-Lefschetz locus and has a natural scheme structure.

An important result by Boij–Migliore–Miro-Roig–Nagel states that for a general Artinian complete intersection of height 3, the non-Lefschetz locus has the expected codimension and the expected degree.

In this talk, we will define in a similar way the non-Lefschetz locus for conics. We say that $C$, a homogeneous polynomial of degree 2, is a Lefschetz conic for $A$ if the multiplication map $\times C$ from $A_i$ to $A_{i+2}$ has maximal rank for each integer $i$. We will show that for a general complete intersection of height 3, the non-Lefschetz locus of conics has the expected codimension as a subscheme of $\mathbb{P}^5$, and that the same does not hold for certain monomial complete intersections.

The study of the non-Lefschetz locus for Artinian complete intersections can be generalized to modules $M = H^1(\mathbb{P}^2, E)$ where $E$ is a vector bundle of rank 2. The non-Lefschetz locus, in this case, is exactly the set of jumping lines of $E$, and

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the expected codimension is achieved under the assumption that $E$ is general.

In the case of conics, the same is not true. The non-Lefschetz locus of conics is a subset of the jumping conics, but it is a proper subset when $E$ is semistable with first Chern class even.