

## Mathematical Computer Science Seminar

### *A Sharp Computational Phase Transition for the Partition Function of the Transverse-Field Ising Model*

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**Abstract:** We study the problem of approximating the partition function of the transverse-field Ising model (TFIM), a widely studied quantum many-body model with important applications in quantum simulation and quantum annealing. Despite its fundamental importance, the algorithmic landscape for computing the TFIM partition function has remained poorly understood beyond restricted parameter regimes. We provide a precise characterization of the temperature regimes in which efficient approximation is possible, establishing a sharp computational phase transition. Let  $J$  denote the symmetric interaction matrix and  $\Delta(J) = \lambda_{\max}(J) - \lambda_{\min}(J)$  denote its spectral width. We show that, for all inverse temperatures  $\beta \in [0, 1/\Delta(J)]$ , there exists an efficient classical randomized algorithm that approximates the partition function  $\text{tr}(e^{-\beta H})$  to within an arbitrarily small multiplicative factor. To obtain this result, we apply the standard Trotter decomposition to map the quantum model to a classical spin system, and then leverage new techniques in Markov chain analysis to derive an efficient algorithm that samples from and computes the partition function of the resulting distribution. This temperature threshold is tight: for  $\beta > 1/\Delta(J)$ , we show that approximating the partition function, even within an exponential factor, is NP-hard and thus is unlikely to admit an efficient classical or quantum algorithm.

Monday, April 27 at 3:00 PM in 1227 SEO

Joint work with Alistair Sinclair.

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