Analysis and Applied Mathematics Seminar

Dynamics of singularities and wavebreaking in 2D hydrodynamics with free surface

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Abstract: 2D hydrodynamics of ideal fluid with free surface is considered. A time-dependent conformal transformation is used which maps a free fluid surface into the real line with fluid domain mapped into the lower complex half-plane. The fluid dynamics is fully characterized by the complex singularities in the upper complex half-plane of the conformal map and the complex velocity. The initially flat surface with the pole in the complex velocity turns over arbitrary small time into the branch cut connecting two square root branch points. Without gravity one of these branch points approaches the fluid surface with the approximate exponential law corresponding to the formation of the fluid jet. The addition of gravity results in wavebreaking in the form of plunging of the jet into the water surface. The use of the additional conformal transformation to resolve the dynamics near branch points allows to analyze wavebreaking in details. The formation of multiple Crapper capillary solutions is observed during overturning of the wave contributing to the turbulence of surface wave. Another possible way for the wavebreaking is the slow increase of Stokes wave amplitude through nonlinear interactions until the limiting Stokes wave forms with subsequent wavebreaking. For non-limiting Stokes wave the only singularity in the physical sheet of Riemann surface is the square-root branch point located. The corresponding branch cut defines the second sheet of the Riemann surface if one crosses the branch cut. The infinite number of pairs of square root singularities is found corresponding to infinite number of non-physical sheets of Riemann surface. Each pair belongs to its own non-physical sheet

Monday, September 18 at 4:00 PM in SEO 636 of Riemann surface. Increase of the steepness of the Stokes wave means that all these singularities simultaneously approach the real line from different sheets of Riemann surface and merge together forming 2/3 power law singularity of the limiting Stokes wave. It is conjectured that non-limiting Stokes wave at the leading order consists of the infinite product of nested square root singularities which form the infinite number of sheets of Riemann surface. The conjecture is also supported by high precision simulations, where a quad (32 digits) and a variable precision (up to 200 digits) were used to reliably recover the structure of square root branch cuts in multiple sheets of Riemann surface.

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