

Logic Seminar

Categoricity and multidimensional diagrams

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Abstract: The categoricity spectrum of a class of structures is the class of cardinals k such that the class has exactly one model of cardinality k up to isomorphism. Shelah's eventual categoricity conjecture is the statement that the categoricity spectrum of an abstract elementary class (AEC) is either bounded or contains an end segment. Roughly, an AEC is a concrete accessible category with directed colimits satisfying a few extra properties. AECs generalize, for example, classes of models of a first-order theory, classes of models in several infinitary logics, and finitely accessible categories with all morphisms monomorphisms.

Shelah's eventual categoricity conjecture is one of the main test questions in non-elementary model theory, but despite forty years and many more pages of partial approximations it remains open. Until now, it was not even known to be consistent with large cardinals. In this talk, I will present a joint work with Saharon Shelah, a proof that Shelah's eventual categoricity conjecture follows from a large cardinal axiom (a proper class of strongly compact cardinals). The main tool is the method of multidimensional diagrams, introduced by Shelah in the eighties and later rediscovered by Zilber in his study of pseudoexponential fields. The main technical result is that (assuming large cardinals), an AEC with unbounded categoricity spectrum is excellent, i.e. any finite diagram of object can be amalgamated in a way that is, in some category-

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theoretic sense, unique. The categoricity conjecture then follows from excellence. The method seems to have a strongly category-theoretic nature, and has many other applications that do not need large cardinals. For example, assuming that cardinal exponentiation is injective (a weakening of the GCH), one gets a full understanding of the categoricity spectrum of AECs with the amalgamation property. Assuming a little bit more than the GCH, the eventual categoricity conjecture also holds for any AEC with no maximal models.

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