Abstract: In this talk I will discuss heat kernel estimates for critical perturbations of non-local operators. To be more precise, let $X$ be the reflected $\alpha$-stable process in the closure of a smooth open set $D$, and $X^D$ the process killed upon exiting $D$. We consider potentials of the form $\kappa(x) = C\delta_D(x)^{-\alpha}$ with positive $C$ and the corresponding Feynman-Kac semigroups. Such potentials do not belong to the Kato class. We obtain sharp two-sided estimates for the heat kernel of the perturbed semigroups. The interior estimates of the heat kernels have the usual $\alpha$-stable form, while the boundary decay is of the form $\delta_D(x)^p$ with non-negative $p \in [\alpha - 1, \alpha)$ depending on the precise value of the constant $C$. Our result recovers the heat kernel estimates of both the censored and the killed stable process in $D$. Analogous estimates are obtained for the heat kernel of the Feynman-Kac semigroup of the $\alpha$-stable process in $\mathbb{R}^d \setminus \{0\}$ through the potential $C|x|^{-\alpha}$.

All estimates are derived from a more general result described as follows: Let $X$ be a Hunt process on a locally compact separable metric space in a strong duality with $\tilde{X}$. Assume that transition densities of $X$ and $\tilde{X}$ are comparable to the function $\tilde{q}(t,x,y)$ defined in terms of the volume of balls and a certain scaling function. For an open set $D$ consider the killed process $X^D$, and a critical smooth measure on $D$ with the corresponding positive additive functional $(A_t)$. We show
that the heat kernel of the Feynman-Kac semigroup of $X^D$ through the multiplicative functional $\exp(-A_t)$ admits the factorization of the form $P_x(\zeta > t)\tilde{P}_y(\zeta > t)\tilde{q}(t,x,y)$.

This is joint work with Soobin Cho, Panki Kim and Zoran Vondracek.