Mathematics, Statistics, and Computer Science **@ UIC**

Statistics and Data Science Seminar

Factorizations and estimates of Dirichlet heat kernels for non-local operators with critical killings

Renming Song (UIUC)

Abstract: In this talk I will discuss heat kernel estimates for critical perturbations of non-local operators. To be more precise, let X be the reflected α-stable process in the closure of a smooth open set D, and X^D the process killed upon exiting D. We consider potentials of the form $\kappa(x) = C\delta_D(x)^{-\alpha}$ with positive C and the corresponding Feynman-Kac semigroups. Such potentials do not belong to the Kato class. We obtain sharp two-sided estimates for the heat kernel of the perturbed semigroups. The interior estimates of the heat kernels have the usual α-stable form, while the boundary decay is of the form $\delta_D(x)^p$ with non-negative $p \in [\alpha - 1, \alpha)$ depending on the precise value of the constant C. Our result recovers the heat kernel estimates of both the censored and the killed stable process in D. Analogous estimates are obtained for the heat kernel of the Feynman-Kac semigroup of the α-stable process in $R^d \setminus \{0\}$ through the potential $C|x|^{-\alpha}$.

All estimates are derived from a more general result described as follows: Let X be a Hunt process on a locally compact separable metric space in a strong duality with $\widehat{X}$. Assume that transition densities of X and $\widehat{X}$ are comparable to the function $\widetilde{q}(t, x, y)$ defined in terms of the volume of balls and a certain scaling function. For an open set D consider the killed process X^D, and a critical smooth measure on D with the corresponding positive additive functional (A_t). We show

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that the heat kernel of the the Feynman-Kac semigroup of X^D through the multiplicative functional $\exp(-A_t)$ admits the factorization of the form $P_x(\zeta > t)\widehat{P}_y(\widehat{\zeta} > t)\widetilde{q}(t, x, y)$.

This is joint work with Soobin Cho, Panki Kim and Zoran Vondracek.

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