# Mathematical Computer Science Seminar

## *Total non-negativity of some combinatorial matrices*

David Galvin (Notre Dame)

**Abstract:** Many combinatorial matrices — such as those of binomial coefficients, Stirling numbers of both kinds, and Lah numbers — are known to be totally non-negative, meaning that all minors (determinants of square submatrices) are non-negative.

The examples noted above can be placed in a common framework: for each one there is a non-decreasing sequence $(a_1, a_2, \ldots)$, and a sequence $(e_1, e_2, \ldots)$, such that the $(m, k)$-entry of the matrix is the coefficient of the polynomial $(x - a_1) \cdots (x - a_k)$ in the expansion of $(x - e_1) \cdots (x - e_m)$ as a linear combination of the polynomials $1, x - a_1, \ldots, (x - a_1) \cdots (x - a_m)$.

I'll discuss this general framework, and for a non-decreasing sequence $(a_1, a_2, \ldots)$ sketch the proof of necessary and sufficient conditions on the sequence $(e_1, e_2, \ldots)$ for the corresponding matrix to be totally non-negative. I'll derive as corollaries the totally non-negativity of matrices of rook numbers of Ferrers boards, and of a family of matrices associated with chordal graphs.

Monday, October 29 at 3:00 PM in 427 SEO