

**Analysis and Applied Mathematics Seminar***Sign-changing solutions of the nonlinear heat equation with positive initial value*

Fred Weissler (Universite de Paris Nord)

**Abstract:** We consider the nonlinear heat equation  $u_t - \Delta u = |u|^\alpha u$  on  $\mathbb{R}^N$ , where  $\alpha > 0$ . It is well known that the Cauchy problem is locally well-posed in a variety of spaces. For instance, for every  $\alpha > 0$ , it is well-posed in the space  $C_0(\mathbb{R}^N)$  of continuous functions that converge to 0 at infinity. It is also well-posed in  $L^p(\mathbb{R}^N)$  for  $p \geq 1$ ,  $p > \frac{N\alpha}{2}$ , but not well-posed in  $L^p$  for  $1 \leq p < \frac{N\alpha}{2}$  if  $\alpha > \frac{2}{N}$ . In particular, for such  $p$  there exist positive initial values  $u_0 \in L^p$  for which there is no local in time positive solution. Also, if one considers the initial value  $u_0(x) = c|x|^{-\frac{2}{\alpha}}$  for all  $x \in \mathbb{R}^N \setminus \{0\}$ , with  $c > 0$ , it is known that if  $c$  is small, there exists a global in time (positive) solution with  $u_0$  as initial value, and in fact this solution is self-similar. On the other hand, if  $c$  is large, there is no local in time positive solution, self-similar or otherwise. We prove that in the range  $0 < \alpha < \frac{4}{N-2}$ , for every  $c > 0$ , there exist infinitely many self-similar solutions to the Cauchy problem with initial value  $u_0(x) = c|x|^{-\frac{2}{\alpha}}$ . Of course, these solutions are all sign-changing if  $c$  is sufficiently large. Also, in the range  $\frac{2}{N} < \alpha < \frac{4}{N-2}$ , we prove the existence of local in time sign-changing solutions for a class of nonnegative initial values  $u_0 \in L^p$ , for  $1 \leq p < \frac{N\alpha}{2}$ , for which no local in time positive solution exists.

This is joint work with T. Cazenave, F. Dickstein and I. Naumkin.

Monday, October 22 at 4:00 PM in 636 SEO