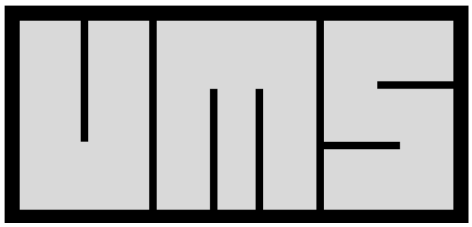
Undergraduate Mathematics Symposium



October 9, 2010
University of Illinois at Chicago

Organized by David Dumas and Steven Hurder

Schedule of Activities

8:15am – 8:55am	Sign-in and coffee in SEO 300
Morning Session – Invited lectures, 50 minutes each, SEO 636	
8:55am	Opening remarks
9:00am	Laura DeMarco (University of Illinois at Chicago) **The Mandelbrot set: what we know today**
10:00am	Sergei Tabachnikov (Pennsylvania State University) **The Tait-Kneser theorem: variations on the theme**
11:00am	George Francis (University of Illinois at Urbana-Champaign) **The Art of Tony Robbin and Visualizing 4 and more Dimensions**
12:00pm – 1:30pm	Lunch in SEO 300
Afternoon Session 1 – Contributed lectures, 20 minutes each, SEO 636	
1:30pm	Steven Pollack (McGill University) **Characterizing steady-state stabilities of a thin-film in a rotating cylinder**
2:00pm	John Goes (University of Illinois at Chicago) **On the Sum of Four Squares in Quadratic Extensions**
2:30pm	Justin Campbell (University of Michigan) **The Dual Space of a p-adic Heisenberg Group**
2:50pm – 3:30pm	Coffee break in SEO 300
Afternoon Session 2 – Contributed lectures, 20 minutes each, SEO 636	
3:30pm	Elizabeth Malczewski (University of Illinois at Chicago) **Using Mathematics in Remote Sensing and in Oil Removal Analysis**
4:00pm	Cyndie Cottrell (McGill University) **Enumerating Perfect Matchings**
4:30pm	Michael Qin (Georgia Institute of Technology) **Orthogonal Polynomials and Admissible Operators in Two Variables**
5:00pm	Alexander Carney (University of Michigan) **Highly noninjective polynomial maps on the rational numbers**

Abstracts of Invited Lectures

Laura DeMarco (University of Illinois at Chicago)

The Mandelbrot set: what we know today

Since the advent of computer graphics in studying mathematics, the Mandelbrot set has been a central object of intrigue and investigation. It plays a fundamental role in the study of dynamical systems. Due to its complicated and beautiful structure, it also appears regularly in popular "fractal" art. In this talk, I will explain what the Mandelbrot set is, what we know about it, and what we don't.

George Francis (University of Illinois at Urbana-Champaign)

The Art of Tony Robbin and Visualizing 4 and more Dimensions

Twice in our history it was art that informed mathematics on the nature of space. In the renaissance, painters discovered the rules of perspective and taught geometers the foundation of projective geometry. Late in the last century, computer graphics transformed how to visualize geometrical phenomena in more dimensions than our senses evolved to perceive. To the amazement of mathematicians, abstractions like hypercubes rotating in 4-space, right-angled dodecahedra tiling hyperbolic 3-space, and spheres turning inside out, could be directly experienced after all. And to the empowerment of artists striving to express their impressions of counter-intuitive physics and multiply over-layered geometrical shapes, computer graphics provides both revelation and design tools.

Tony Robbin, more significantly than others, plumbed the depth of this new medium in the service of art. Mastering the requisite sciences to make his work true and persuasive, he broke through the barrier separating fine artist and mathematician. This brief essay is how one of the latter fell under Robbin's spell, and why it matters.

This presentation is about a collaboration of Robbin and Francis to create a virtual quasicrystal installation in the CAVE. Already five participants in a my research experience for undergraduates (REU) have programmed parts of of this project.

Sergei Tabachnikov (Pennsylvania State University)

The Tait-Kneser Theorem: Variations on the theme

The Tait-Kneser theorem states that the osculating circles of a plane arc with monotone curvature are pairwise nested. I shall explain this theorem and discuss its variants. I shall connect the Tait-Kneser theorem with the 4-vertex theorem and its variants (a plane oval has at least four curvature extrema). I will also draw (conjectural) connection to various versions of the Sylvester problem (given a finite set of points in the plane, not all collinear, there is a line through exactly two of these points).

Abstracts of Contributed Lectures

Justin Campbell (University of Michigan)

The Dual Space of a p-adic Heisenberg Group

Francois Rodier proved that it is possible to view smooth representations of certain totally disconnected abelian groups (e.g. additive groups of finite-dimensional p-adic vector spaces) as sheaves of vector spaces on the Pontryagin dual group. There is an appropriate notion of dual space for nonabelian totally disconnected groups (e.g. p-adic unipotent groups) which generalizes Pontryagin's construction, but it includes representations which are not one-dimensional and does not carry a natural group structure. In my REU at the University of Michigan, we proved that in this generality, smooth representations still share many properties with sheaves on the dual space. In particular, we used spectral decomposition to give a sheaf-theoretic description of smooth representations of p-adic Heisenberg groups.

Alexander Carney (University of Michigan)

Highly noninjective polynomial maps on the rational numbers

What polynomials are maximally noninjective over the rational numbers? More specifically, for what polynomials f in Q[x] are there infinitely many pairs (a,b) of distinct rational numbers such that f(a)=f(b)? The Mordell Conjecture shows that this question can be related to the genus of the algebraic curve f(X)-f(Y). We will show how to compute this genus and look at methods used to classify all polynomials such that f(X)-f(Y) has an irreducible factor with genus zero or one. This work is the result of a REU at the University of Michigan, summer 2010.

Cyndie Cottrell (McGill University)

Enumerating Perfect Matchings

We will begin with an introduction to the basics of graph theory and perfect matchings, and proceed to discuss the results of undergraduate research done this summer under the support of an ISM Summer Scholarship in Montreal at McGill University. By splitting the graph into finite subgraphs and applying a recently developed domino shuffling, we will find an enumeration of the perfect matchings on the dual graph of the hexagonal lattice.

John Goes (University of Illinois at Chicago)

On the Sum of Four Squares in Quadratic Extensions

A classic proof of Lagrange demonstrates that every integer can be expressed as the sum of four integer squares. For example $10=2^2+2^2+1^2+1^2$. These representations are not unique, since for example we also have $10=3^2+1^2+0^2+0^2$. Jacobi exhibited a function $r_4(n)$ which yields the number of such representations for a given integer n including those obtained by reordering terms. In recent decades, attempts have been made to generalize this result to other number fields. In $Q[\sqrt{2}]$, for example, $10=(1+\sqrt{2})^2+(1-\sqrt{2})^2+2^2+0^2$. Analogues to Jacobi's four-square theorem have been extended variously by Cohn, Grotsky and Deutsch to integer rings of the quadratic fields $Q[\sqrt{2}]$, $Q[\sqrt{3}]$,$Q[\sqrt{5}]$. We consider the more problematic extension field $Q[\sqrt{7}]$ and exhibit its Jacobi function. Other extensions are considered and new questions are posed.

Abstracts of Contributed Lectures (continued)

Elizabeth Malczewski (University of Illinois at Chicago)

Using Mathematics in Remote Sensing and in Oil Removal Analysis

At NASA Stennis Space (Summer 2010), I used mathematics, the math software MATLAB, and other remote sensing products to study changes in Louisiana coastal marshes, as well as to predict whether Barataria Bay, LA would be suitable for the oil removal technique, in-situ burning. My research includes the use of matrices, interpolation, and other mathematical principles.

Steven Pollack (McGill University)

Characterizing steady-state stabilities of a thin-film in a rotating cylinder

My REU focused on steady-state solutions to the thin-film ODE modeling the rimming flow of a thin-film in a rotating cylinder; Given parameters q, ε, and the mass of the film, the natural question was how each of these three values were inter-related. The emphasis of the talk will be on certain challenges associated with building the (q,ε,M) surface, and the subsequent trends obtained from this surface.

Michael Qin (Georgia Institute of Technology)

Orthogonal Polynomials and Admissible Operators in Two Variables

This summer REU project attempts to describe a family of 2 dimensional orthogonal polynomials. It is possible that a more general class than the Askey-Wilson scheme exists; specifically, there may exist a linear operator whose eigenfunctions form an orthogonal polynomial sequence. The lecture will give an overview of this approach, focusing on two necessary and sufficient conditions for the operator. The first condition can be proven using only linear algebra and calculus, but a complete solution to the second condition has so far resisted brute force methods.