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Kauffman, Louis H.; **Banchoff, Thomas F.**
Immersions and mod-2 quadratic forms.
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Let "surface" mean a compact Hausdorff differentiable 2-manifold with or without boundary. The authors begin with a study of the topology of differentiable immersions of orientable surfaces into the 2-sphere S^2. Then the authors show how this leads to quadratic forms and discuss them in more detail. By doing this, the authors obtain geometric proofs and interpretations for the basic algebraic identities which underlie the theory of mod 2 quadratic forms.

Letting $C(M)$ denote the set of differentiably imbedded curves α on an orientable surface M, for each immersion $f\colon M\to S^2$, a function $N(f)\colon C(M)\to Z_2$ is introduced by the number of normal crossings of $f\circ\alpha$ reduced mod 2. $N(f)$ measures how curves on M are immersed into S^2. Then an invariant $B(f)\in Z$ is defined by the total number of boundary curves $C\subset M$ which satisfy $N(f)(C)=1$. The authors use $B(f)$ to classify immersions of punctured disks up to an equivalence called image homotopy, using basic homotopies (handle sliding and permutation). The authors also show how $N(f)$ leads to a quadratic form $q(f)\colon H_1(M;Z_2)\to Z_2$ and how certain homotopies of immersions correspond to isomorphisms of quadratic forms. Then the authors explain the classification of mod 2 quadratic forms and obtain classification theorem of immersions: Orientation preserving immersions $f,g\colon M\to S^2$ are image homotopic if and only if $B(f)=B(g)$ and $q(f)$ is isomorphic to $q(g)$. {Arguments of all parts in the paper are very intuitive using many figures.}

Reviewed by *H. Suzuki*