

**Quiz5 - Math 313 - Fall 2014**

1. (a) Prove that the cardinality of the set  $P(X)$  of subsets of a set  $X$  is greater than the cardinality of  $X$ .

(b) True or False: Every subset of the Real Numbers, other than the Real Numbers themselves, is countable.

(c) Let

$$\lambda = .d_1d_2d_3 \cdots$$

where  $d_n = 0$  if  $n$  is not a power of 2 and  $d_n = 1$  if  $n$  is a power of 2. Thus

$$\lambda = .0101000100000001000000000000001 \cdots.$$

Regarding  $\lambda$  as a real number represented by an infinite decimal, prove that  $\lambda$  is irrational.

(d) Let

$$\rho = .d_1d_2d_3 \cdots$$

where  $d_n = 0$  if  $n$  is a prime number and  $d_n = 1$  if  $n$  is not a prime number. Thus

$$\rho = .01101010001010001 \cdots.$$

Regarding  $\rho$  as an infinite decimal, do you think that  $\rho$  is a rational or irrational number? We do not expect you to prove which of these possibilities is the case, but think about it and give your reasons for thinking one way or the other.