Quiz6 - Math 313 **- Fall** 2014

Let $\mathcal{R}$ denote the real numbers. A function $f : \mathcal{R} \longrightarrow \mathcal{R}$ is said to be continuous at a point $a \in \mathcal{R}$ if $lim_{x \longrightarrow a} f(x) = f(a)$. In detail, this means that the above limit exists and is equal to $f(a)$. In even more detail it means that given an $\epsilon > 0$ there exists a $\delta > 0$ such that whenever $0 < |x - a| < \delta$, then $|f(x) - f(a)| < \epsilon$.

The function f is said to be continuous on a subset S of $\mathcal{R}$ if it is continous at every point of S. Thus f is continuous on $\mathcal{R}$ if it is continuous at every point of $\mathcal{R}$.

1. (a) Show that $f(x) = x^2$ is continuous at every point in $\mathcal{R}$.

   (b) Define $g : \mathcal{R} \longrightarrow \mathcal{R}$ by $g(x) = x/|x|$ when $x \neq 0$ and $g(0) = 0$. Show that g is continuous at every point in $\mathcal{R}$ except $a = 0$. Describe how the definition of continuity fails for g at $a = 0$.

   (c) Let $f : \mathcal{R} \longrightarrow \mathcal{R}$ be given to be continuous on all of $\mathcal{R}$ and also let it be given that $f(x) > x$ for all $x \geq 0$. Let x_0 be a chosen non-negative real number. Define a sequence of real numbers via

$$x_{n+1} = f(x_n).$$

   Show that this sequence must diverge to positive infinity. (Hint: Show that if (x_n) does not diverge then it has a limit x such that $f(x) = x$. Explain why this gives a contradiction, and hence a proof of divergence for the series.)