

**Math 215: Introduction to Advanced Mathematics**  
Midterm I–Study Guide

- The midterm exam will be on Friday October 6. The exam will cover chapters 1–6.
- The course web page contains a week-by-week syllabus  
<http://www.math.uic.edu/~marker/math215/wtow.html>  
and a list of key concepts  
<http://www.math.uic.edu/~marker/math215/concepts.html>  
that gives a more detailed description of the material you are responsible for.
- One good way to study is to work on the sample problems suggested on the course web page.
- If I ask you to prove something from the axioms for ordered fields, I will provide you with a copy of the axioms.

**Sample Exam**

- 1) Consider the statement: if  $a > b$ , then  $f(a) > f(b)$ .
  - a) What is the contrapositive of this statement?
  - b) What is the converse of this statement?
  - c) What is the negation of this statement.

- 2) a) Determine the truth table for

$$(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$$

- b) Find a simpler statement equivalent to  $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$ .
- 3) Prove from the axioms for ordered fields that if  $0 < x < y$ , then  $x^3 < y^3$ .
- 4) Prove that if  $a^2 \geq 7a$ , then  $a \leq 0$  or  $a \geq 7$ .
- 5) Prove that

$$\prod_{i=2}^n \left(1 - \frac{1}{i^2}\right) = \frac{n+1}{2n}$$

for all  $n \geq 2$ .

- 6) Prove that  $A \subseteq B$  if and only if  $A \cup B \subseteq B$ .