Math 215: Introduction to Advanced Mathematics
Midterm II–Study Guide

• The second midterm exam will be on Friday November 17. The exam will cover chapters 6–10 + 12 (up to and including pg 148).

• The course web page contains a week-by-week syllabus
`http://www.math.uic.edu/~marker/math215/wtow.html`
and a list of key concpets
`http://www.math.uic.edu/~marker/math215/concepts.html`
that gives a more detailed description of the material you are responsible for.

• One good way to study is to work on the sample problems suggested on the course web page.

Sample Questions[1]

1) Define the following concepts:
a) The sequence $(a_n)_{n=1}^{\infty}$ converges to a.
b) $f : X \to Y$ is injective.

2) Negate the following statements
a) $\forall x \in \mathbb{R} \exists y \in \mathbb{R}\ f(y) = x$
b) $\exists x \in A \forall y \in B\ (f(y) = x \text{ or } g(y) = x)$.

3) Let $A = \{1, 2, 3, 4\}$, let $B = \{1, 2, 3, 4, 5\}$ and define $f : A \to B$ by

x	$f(x)$
1	2
2	2
3	3
4	5

a) What is $\overrightarrow{f}(\{1, 2\})$?
b) What is $\overleftarrow{f}(\{2, 4, 5\})$?

[1] This sample is consdierably longer than the midterm will be.

4) Decide if each of the following statement is TRUE or FALSE. If FALSE, give an example showing it is FALSE.

a) Suppose $f : X \to Y$ and $g : Y \to X$ such that $g \circ f = I_X$. Then g is the inverse of f.

b) For all sets A and B, if $A \subseteq B$, then $B^c \subseteq A^c$.

c) For all sets A and B, $|A \cup B| = |A| + |B|$

d) $\forall x \in \mathbb{Z} \exists y \in \mathbb{Z}\ y < x$.

d) If $f : X \to Y$ and $A, B \subseteq X$ and $A \subseteq B$, then $\overrightarrow{f}(A) \subseteq \overrightarrow{f}(B)$.

6) Suppose $f : X \to Y$ is a bijection. Prove that $\overrightarrow{f} : \mathcal{P}(X) \to \mathcal{P}(Y)$ is a bijection.

7) Suppose $m, n > 0$. Let $F = \{f : \mathbb{N}_m \to \mathbb{N}_n \text{ such that } f(1) = 1\}$.

a) Prove there is a bijection between F and $\mathcal{F}(\mathbb{N}_{m-1}, \mathbb{N}_n)$.

b) What is $|F|$?

8) Using the definition of convergence, prove that the sequence $(\frac{1}{\ln(n+1)})_{n=1}^{\infty}$ converges to 0.