Math 435 Number Theory I
Problem Set 7

Due: Friday October 21:

1) Find all solutions to $X^2 + X + 223 \equiv 0 \pmod{243}$. Note $243 = 3^5$.

2) Let p be prime and let $f(X,Y) \in \mathbb{Z}[X,Y]$. Suppose $a, b \in \mathbb{Z}$, $f(a,b) \equiv 0 \pmod{p}$ and $\frac{\partial f}{\partial X}(a,b) \not\equiv 0 \pmod{p}$. Prove that $f(X,Y) \equiv 0 \pmod{p^n}$ has a solution for all $n \geq 1$. [Hint: Consider the polynomial $g(X) = f(X,b)$.]

3) a) Calculate $\phi(100)$.

b) Find the last two decimal digits of 3^{403}.

4) Suppose $1 = b_1 < b_2 < \ldots < b_{\phi(n)}$ are the integers between 1 and n that are relatively prime to n.

a) Show that $B = \displaystyle\prod_{i=1}^{\phi(n)} b_i \equiv \pm 1 \pmod{n}$.

b) (Bonus 5pt) Characterize the n for which $B \equiv -1$.