Math 435 Number Theory I
Sample Problems–Midterm I

Midterm 1 will cover chapters 1–3 of Jones and Jones and the supplementary material on Pythagorean triples.

The questions below are representative of the type of questions I might ask. (Note: This sample is, most likely, longer than the exam will be.)

1) State the Fundamental Theorem of Arithmetic.

2) Define the following concepts.
a) n is prime
b) $a \equiv b \pmod{n}$.

3) Use the Euclidean algorithm to find $\gcd(665, 114)$.

4) Decide if each of the following equations have integer solutions. If they can, find the general solution.
a) $665x + 114y = 9$
b) $665x + 114y = 38$

5) Find all solutions to the system of equations

$$x \equiv 3 \pmod{5}, x \equiv 2 \pmod{11} \text{ and } x \equiv 1 \pmod{3}$$

6) Find all congruence classes mod 36 solving $6x \equiv 3 \pmod{36}$.

7) Decide if the following statements are TRUE or FALSE. If FALSE, give an example showing why they are FALSE.
a) If $\gcd(a, b) = 1$, $a|n$ and $b|n$, then $ab|n$.
b) There is a unique congruence class mod 60 solving the system of equaitons

$$x \equiv 3 \pmod{10}, x \equiv 5 \pmod{12}, x \equiv 2 \pmod{15}$$

c) If $ab \equiv ac \pmod{n}$ and $a \not\equiv 0 \pmod{n}$, then $b \equiv c \pmod{n}$.

8) Suppose $\gcd(a, b) = d$ and $\gcd(a, n) = 1$. Prove that $a \equiv b \pmod{n}$ if and only if $\frac{a}{d} \equiv \frac{b}{d} \pmod{n}$.

9) Prove there are is no integer x with $x^3 - x^2 + 3x = 16$ [Hint: Consider congruences mod some small primes.]

10) Suppose $m > 1$, $a \in \mathbb{N}$ and $a^m - 1$ is prime. Prove that $a = 2$ and m is prime.